Bass in Small Rooms

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1. Introduction

A common and pervasive myth about loudspeakers goes thus: it is impossible to produce low frequency bass in a small room. The reason, stated almost axiomatically, is because the room is too small to hold the long wavelengths.

That this is a myth without physical foundation is easy to demonstrate on several fronts. For one, if the axiomatic basis were to hold, then by the same axiom, we should not be able to hear any frequencies below several kilohertz, because these same large wavelengths could not fit within the small confines of the ear canal and thus could not excite the eardrum into motion. We can demonstrate the fallacy of this argument more analytically, and use an acoustic device as an existence proof that one can support low frequencies in small enclosures.

2. A Case Study: The Pistonphone Acoustic Calibrator

The erstwhile and clever Danish Acoustics instrument manufacturer, Brüel & Kjær, produces a device, the type 4220 PistonPhone, which is used in the absolute calibration of microphones. The operating principle behind the Pistonphone is conceptually simple: a sealed enclosure of volume V whose volume is varied periodically by a change in volume Δv will produce a sound level of:

$$P = \gamma \times P_0 \times \frac{\Delta v}{V} \text{ dynes / cm}^2$$

where:

γ = the ratio of specific heats for the gas in the enclosure. For air at 20°C and at 1 atmosphere, $\gamma = 1.402$.

P_0 = atmospheric pressure, 1.0133×10^6 dynes / cm^2

Δv = the change in volume of the enclosure

V = the total volume of the enclosure

The 4220 pistonphone is implemented as two opposed cam-driven pistons, approximately 4 mm in diameter that work into a closed volume of approximately 19 cubic centimeters (about the volume of an average ice cube) . The pistons are driven each by a 4-lobed cam with a sinusoidal profile by a small speed-governed motor which revolves at 3750 RPM. The result is that the volume of the chamber varies sinusoidally at a rate of 250 Hz (3750/60×4).

Now, then chamber itself is quite small. Brüel & Kjær claims, as mentioned, that the “coupling chamber” has a volume of some 19 cm^3. Its linear dimensions are that of a cylinder approximately 3.19 cm in diameter with a depth of 2.63 cm, Within that volume is the piston housing itself, which occupies a small portion of the total volume. The cam has a maximum radius of about 0.55 cm and a minimum radius of 0.5 cm. This means the total stroke of each of the two pistons is 0.05 cm. The resulting displacement, or change in volume, is then

$$\Delta v = \pi \frac{d^2}{4} s$$

where d is the diameter of the piston, and s is the stroke of the piston. In the case of the measurements here, we get:

$$\Delta v = \pi \frac{(4 \times 10^{-3}\,\text{m})^2}{4} \times 5 \times 10^{-4}\,\text{m}$$

$$\Delta v = \pi \frac{16 \times 10^{-6}\,\text{m}^2}{4} \times 5 \times 10^{-4}\,\text{m}$$

$$\Delta v = 6.28 \times 10^{-9}\,\text{m}^3$$

Six *billionths* of a cubic meter is a small volume, to be sure.

But it is that six billionths of a cubic meter change that produces the 124 dB sound pressure level. Looking at it another way, varying the volume of a sealed enclosure by:

$$\frac{6.28 \times 10^{-9}\,\text{m}^3}{1.9 \times 10^{-5}\,\text{m}^3} = 0.00033$$

a mere 0.03% change in volume! Putting it all together, we come up with a corresponding change in pressure of:

$$P = 1.402 \times 1.0133 \times 10^{6}\, dynes/cm^{2} \times \frac{6.28 \times 10^{-9}\, m^{2}}{1.9 \times 10^{-5}\, m^{3}} = 470\, dynes/cm^{2}$$

Sound pressure level in dB S.P.L. is calculated as:

$$\text{dB S.P.L.} = 20 \log_{10} \frac{\text{P}}{\text{P}_{\text{REF}}}$$

where P_{REF} is the 0 dB S.P.L. reference sound pressure level corresponding to 0.0002 dynes/cm^2. In this case:

$$\text{dB S.P.L.} = 20 \log_{10} \frac{470\ \text{dynes}/\text{cm}^{2}}{0.0002\ \text{dynes}/\text{cm}^{2}}$$

This corresponds to a sound pressure level of about 127 dB S.P.L.. This is the peak sound level produced. Since the cam profile and thus the waveform produced is sinusoidal, the equivalent RMS sound pressure level is 3 dB less, or 124 dB, essentially what Brüel & Kjær claim.

The difficulty with reconciling this result with the notion that you must be able to fit a wavelength within the chamber is that at 250 Hz, the wavelength is about 1.36 meters (some 4 ½ feet), while the maximum linear dimension of the chamber is a mere 0.03 meters (1 ¼ inch). According to the oft-held theory stated above, this chamber is too small. Not by a little, but by a factor of over 40! The oft-held theory suggests that such a small chamber could not support any bass less than 10 kHz.

Not only that, but by simply varying the drive voltage to the motor, we can change the rotation speed of the motor. When we slow the motor down so that it produces frequencies below 250 Hz, we noticed something very interesting: the sound pressure level actually measured in that little chamber *does not change*! Even lowering the speed of the motor corresponding to a frequency of 15 Hz, we find that the sound pressure is the same 124 dB. Indeed, we can continue lowering the frequency and, as long as the response of the microphone itself holds out, we see no reduction in the sound pressure level until we get to a frequency lower enough where tiny leaks in the chamber start to dominate. Seal the chamber tight enough, and these leaks may not be significant until below a fraction of a Hertz.

3. What is “Sound?”

For the purposes of our analysis, sound is the physical phenomenon of varying air pressure to a sufficient amplitude and at an appropriate frequency such that we can detect it using our ears. Make the pressure different on one side of the eardrum vs the other, the ear drum moves in response, and, eventually, this fact is communicated to the brain and, maybe, interpreted. All that is needed is a change in pressure between the outer ear and the inner ear. There are limits to the frequency at which the external pressure changes result in differential pressure between the inner and outer ear. Built in to the ear is its own low-frequency “leak,” the Eustachian tube. This small tube communicates air between the chamber behind the eardrum and the back of the throat. One of it’s purpose is to make sure that very low frequency changes in pressure do not cause movement of the eardrum. These low-frequency pressure changes would, for example, include such things as normal variations in atmospheric pressure. It’s reasonable to assume that the Eustachian tube is an evolutionary response to such pressure variations: if the tube is blocked, it can lead to ear pain due to pressure variations.

Conveniently, evolution has inadvertently foreseen the role of the Eustachian tube in helping us cope with modern-day airplane travel, which cause rather large external pressure changes in a relatively short amount of time. This is why flying can be very uncomfortable if you have a head cold: inflammation and mucus can plug the Eustachian tube, preventing pressure equalization

That being said, any mechanism that results a time-varying change in pressure between the outer and inner ear of a sufficiently high frequency will result in the perception of sound. It doesn’t make any difference whether that pressure change was due to the passing of a wave of pressure higher or lower than ambient, or simply a raising or lowering of the overall pressure in the surrounding airspace: as far as the ear is concerned (at low frequencies, at least), they are indistinguishable. This is why headphones, for example, can convey real low-frequency information: they simply cause a difference in pressure between the outer and inner ear.

4. Bass in Small Rooms

So, now we have an actual physical basis refuting the notion that one cannot have low bass in small rooms. Can we show what, in fact, the effect is in reality?

Yes, however, there are some assumptions to considered. Note that the above discussion, especially the pistonphone analysis, assumes:

1. That the "room" is sealed such that any leaks only affect the very low frequency behavior of the enclosure. The smaller the leaks, the longer it takes for the room to equalize (the longer the time constant of the room), the lower the cutoff frequency,
2. That the size of the enclosure is, indeed, substantially smaller than the wavelength we are attempting to reproduce

This last requirement is very important, because in order for the phenomenon to work as it does in the case of the pistonphone or a headphone, we must be pressurizing the room as a whole. When the wavelengths become small compared to the size of the room, then we have some areas where the pressure is higher than others, the wave behavior of the room begins to dominate, the room begins to look more and more like an very large (maybe even "infinite") space, and overall pressurization is not possible.

What does this mean in the context of our speakers and our rooms? We can, in fact, draw some interesting assertions about achieving low-bas performance with this knowledge in hand.

Remember that the sound pressure level, where the room is reasonably tightly sealed and the wavelengths are long compared to the room dimensions, is dependent solely on the change in volume of the room (given normal air at normal pressure and temperature). That means that we can fairly easily calculate the sound level possible under these conditions from a pair of woofers of a given diameter and a given excursion capability. Simply, the change in volume is the so-called total displacement volume V_D of the woofers: that's the product of the linear excursion X_{MAX} and the emissive diameter S_D of the drivers. Plug those numbers into the equation above, along with the room volume and you now can determine the sound pressure level capabilities of the speaker system.

For example, imagine a 12" woofer with an peak XMAX of ½". In metric terms, that results in an displacement volume of about $6 \times 10^{-4}\,\mathrm{m}^3$. Now, imagine a room with a volume of some 31 cubic meters (corresponding to a room with dimensions of 14' by 10' by 8' high). Using our equation above, one such woofer is capable of generating a sound pressure level of:

$$\mathrm{P} \;=\; \gamma \times \mathrm{P}_0 \times \frac{\Delta \mathrm{v}}{\mathrm{V}} \;\; \mathrm{dynes} / \mathrm{cm}^2$$

$$P \;=\; 1.402 \times 1.0133 \times 10^6 \times \frac{6 \times 10^{-4}\, m^3}{3.1 \times 10^1\, m}$$
$$= 27.5 dynes / cm^2$$

Referenced to 0.0002 dynes/cm2, this corresponds to a sound pressure level of:

$$S.P.L. = 20 \log_{10} \frac{27.5 dyne / cm^2}{0.0002 dyne / cm^2}$$
$$= 20 \log_{10} 1.38 \times 10^5$$
$$= 102.7 dB$$

103 dB SPL is a respectable sound level, especially at such low frequencies from a single woofer. For two such woofers operating in phase, the limit resulting from the doubling in displacement volume would be

$$P \;=\; 1.402 \times 1.0133 \times 10^6 \times \frac{12 \times 10^{-4}\, m^3}{3.1 \times 10^1\, m}$$
$$= 55 dynes / cm^2$$

$$S.P.L. = 20 \log_{10} \frac{55 dyne / cm^2}{0.0002 dyne / cm^2}$$
$$= 20 \log_{10} 2.75 \times 10^5$$
$$= 108.8 dB$$

Note that these figures are the peak sound levels, the RMS sound levels will be about 3 dB lower than these numbers. But that still leaves a respectable 106 dB capability.

The implications of this are important: below some frequency determined by the relationship between the wavelength at that frequency and the maximum dimensions of the room, the bass frequency performance of the speaker and room, considered as a system, is dependent only on the ability of the cone to compress the air, and is independent of frequency! The limit to producing sound level is simply determined by the ratio of the displacement volume of the woofer and the volume of the room.

5. Direct Radiator Loudspeaker Behavior

Contrast this to the behavior required of the woofer when the wavelengths are small compared to the room dimensions, or the so-called "free-field" conditions. Here, the excursion of the cone must go as the inverse square of the frequency in order to maintain a flat frequency response. This condition is satisfied mechanically by drivers operating in the so-called mass-controlled region of operation, above fundamental mechanical resonance. Below resonance, the system is operating in the so-called stiffness controlled region, and the excursion no longer goes as the inverse square of the frequency rather the excursion is constant with frequency. This is precisely the behavior needed for flat frequency response in the pressurized room case!

It would seem, then, that what we need to achieve is a system where the room dimensions and system resonance are coordinated in a way such that above system resonance, the system sees near free-field conditions, and below resonance, it is working to pressurize the room as a whole. That directly suggest that there is a relationship between room size and system resonance for the extended low-frequency performance we are trying to achieve. But what are the room dimension requirements?

The general assumption is made that the largest room dimension must be smaller than ¼ the wavelength of the frequency for the pressure conditions to hold (and again, this also assumes that the room is reasonably well sealed such that the cutoff due to leakage is at a significantly lower frequency). That means, for example, for a system with a 30 Hz system resonance, that the room's largest dimension cannot exceed ¼ wavelength at 30 Hz. That wavelength is ¼ of 342 m/sec/30 Hz = 2.85 meters, or 9 ½ feet. Now, 9 ½ feet is not a big room (though it is a big car!). Consider, instead, a system cutoff of 20 Hz, where the required room size is now relaxed to 4.3 meters or 14 feet.

Another caveat is a corollary of the requirement room cannot have any significant leaks: the speaker enclosure itself cannot "leak" back into the room we are attempting to pressurize. This means that sealed box/acoustic suspension systems will work, but bass reflex or dipole system will not. The rear pressurization of these speakers is communicated directly into the room, and the speaker is not capable of generating the required differential pressures below the vent/enclosure resonant frequency. This is because the rear "chamber" is communicating directly to the room in such speakers at very low frequencies.

Now, lest we leap at the assumption that the ultimate capabilities of the system are without limit, at least as far as low frequency limit is concerned, let's look at some of the other assumptions and see whether they are true.

Certainly, there is the limit imposed by the basic low-frequency time constant of the room and whatever leaks may be present in the room. However, another assumption is that the basic mechanism of how the air itself operates is unchanged at arbitrarily low frequencies. It is assumed that compressions and expansions of the kind normally encountered in sound are adiabatic in nature. This means that the total energy of the system remains constant. The effect of this is that the temperature of the air varies as an inverse function of the volume, and as a direct function of the pressure. Compress the air, raise the pressure, and the temperature increases. Lower the pressure, and it decreases.

The adiabatic behavior of air is certainly true at normal audio frequencies. However, at very low frequencies, this may not be the case, because there can exists mechanisms that will tend to equalize the temperature and attempt to bring the system back to thermal equilibrium. Imagine, for example, the pressure varying slowly enough that the temperature of the walls tends to start to control the temperature of the air. Of course, this is not going to happen until we get to a very low frequency, but the example does illustrate that then technique is not without some fundamental limits in the very nature of the mechanisms behind sound.

6. Caveats

In addition to meeting the fundamental requirement outlined above, there are other considerations at work. Well above both resonance and the frequency where the wavelengths are proximal to the room size, then speaker is operating under semi-free-field conditions, and the $1/f^2$ excursion behavior of the driver ensures flat frequency response into the room. Below system resonance and below the frequency where the wavelength is proximal to room size, the pressurization effect and the frequency-independent excursion behavior of the woofer work to ensure flat frequency response into the room.

It is the region between these two that becomes problematic. This is the region where the response of the speaker/room is often dominated by narrow-band, standing-wave induced phenomenon. It is the frequency where the wavelengths are the same size (roughly) as the room, and the speaker/room system is no longer behaving as either a free-field system or a pressurized system. Rather, it is a resonant system.

7. Conclusion

We have demonstrated that it is possible to generate audible very low frequency sound in enclosures of arbitrarily small volume, utilizing the ability of a loudspeaker or other transducer to produce overall pressure changes in the room. The behavior requires that the room dimensions be smaller than the wavelength of sound being produced and that the time constant of the room resulting from leaks be large compared to the frequency being produced. We've also shown that it is possible to integrate the response of the system at higher frequencies with that at lower be considering the relative placement of room size and system resonance/cutoff frequency using sealed box systems.

Whether such information has practical application is another matter. Given practical sealed box systems (practical defined by reasonable enclosure volume and system efficiency needed to achieve a low enough cutoff frequency), the limitations then imposed on room size may be too restrictive to be usable.

However, given the small dimensions in automobiles, it is possible to utilize this low-frequency loudspeaker and "room" pressurization behavior to effectively extend the response of the system to very low frequencies, now limited only by the time constant of the leaks in the car. This is one reason why the auto sound industry has, even though possibly inadvertently, achieved the ability to produce phenomenonally high sound pressure levels at very low frequencies. The oft-annoying, oft obnoxious car whose incessant booming can be heard blocks away, while not a good existence proof of the concept, is compelling evidence of the efficacy. The only consolation is that as bad as it sounds to us hapless pedestrians, the result is far worse for the occupants of the vehicle, as we are lucky enough to be in the free field response, while they are stuck in the pressurized response.