On The Mechanics of Tone Arms

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Wherein we explore some of the basic physical mechanics of tone arms.

1 INTRODUCTION

"Mechanics" is a branch of physics that explores the behavior and analysis of moving masses and the forces associated with these moving masses. Applying the analysis of mechanics to tone arms is something that is an essential part of tone arm design, but is oft neglected outside of the design process. This has lead, among other things, to a number of myths about how tone arms behave and what constitutes the crucial factors affecting their behavior.

Here are some examples:

- When calculating the mass of a tone arm, you must add the tracking force.
- To reduce the effective mass, you should make the counterweight less massive.

In much of the analysis below, we make some simplifying assumptions. For example, when we generally speak of a mass, we assume, for simplicity, that all of the mass is concentrated in one point, or at least in a region that is very small compared to the other dimensions of the problem. We'll also assume that all structures are rigid and all bearings are without friction. In most cases, the difference that result from these assumptions lead to analysis results that do not appreciably differ from physical reality, and are thus good models of that reality.

In other case, we will include the corrections due to masses actually occupying a large extent, and the effects of friction, because they can affect the behavior of the systems we are analyzing in non-trivial ways.

I, in no way, intended this to be a comprehensive article on tone arm and turntable design. There are plenty of sources, many of them excellent, on various aspects of tone arm design. There are many articles to be had on design factors to minimize tracking error and other aspects of geometry, RIAA preamplifier design, tone arm damping and more.

Nor do I intend to preach a particular gospel even in the limited scope of the present topic: I am, for example, advocating neither high arm mass or low arm mass as being superior.

Rather, I am presenting the basic physics behind why tone arms have the mass that they do, with the hope that armed with this information, the reader can better understand the factors involved. Nowhere, necessarily, is there any information to be found here on *how* to design the very best arm: rather, what is presented is the fundamentals needed to do that job. And also I hope that there's enough here to allow the reader to separate the important wheat from the advertising and mythological chaff that oft surrounds discussions on LP playback principals.

2 A QUESTION OF UNITS

Throughout this discussion, we'll be using the metric system of units. More specifically, we'll be using the *cgs* system, standing for ***c***entimeters, ***g***rams and ***s***econds. The metric system of units allows for easy and consistent units calculations. On the other hand, the normal "imperial" units have little logical relation to one another and are, indeed, a source of great confusion.

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The basic unit of length or distance is the centimeter, abbreviated as cm. A centimeter is a little more than 3/8 of an inch. From that, area is measured in square centimeters and volume in cubic centimeters.

The basic unit of mass, a measurement of the amount of matter, is gram. There are about 28 grams in a dry ounce. We might find it curious that a single cubic centimeter of water has a mass of almost precisely 1 gram. In fact, a gram was initially defined as the mass of 1 cubic centimeter of water at its maximum density, at about 2 degrees Celsius.

Of course, we know of the unit of time, the second.

From these basic units, we can combine them to arrive at the other units that we'll be needing.

When an object is moving, or changing its position, we measure that in terms of distance per time, or centimeters per second, or cm/s.

On the other hand, then the velocity of an object changes, it is said to accelerate, and the acceleration is measured in terms of the change in speed per time, in $\mathrm{cm/s/s}$, or more conventionally, $\mathrm{cm/s^2}$. Earth's gravity acts to accelerate objects at the same rate, an acceleration of $980\ \mathrm{cm/s^2}$ often referred to with the symbol *g*.

A force acts upon a mass to cause it to change speed, or accelerate. Specifically, from Newton, we have the :

$$F = m \times a$$

Force is measured in units of gram centimeters per second squared, or $\mathrm{g\,cm/s^2}$. This is a rather cumbersome unit, so we use the units of dynes instead.

Before we go further, it's necessary to correct a long-standing misconception about tracking force. Since tone-arm time immemorial, tracking "force" has been described in terms of grams. Unfortunately, grams is a unit of mass, not of force, and the use of units of mass to describe force is inappropriate and leads to confusion and incorrect assumptions.

When it is said that an arm has been set up with a "tracking force of 1 gram," what is really meant is that a a force is set equivalent to that of 1 gram under the influence of gravity. Again, remember that:

$$F = m\,a$$

The acceleration of gravity is $980\ \mathrm{cm/sec^2}$ so that 1 gram exerts a force of:

$$F = 1\mathrm{g} \times 980\ \mathrm{cm/sec^2} = 980\ \mathrm{dynes}$$

So, it is more correct to talk of a tracking force of 980 dynes rather than 1 gram. Regrettably, the improper use of grams as a measure of tracking force has become all-pervasive and is hard to shake. I would suggest that we might want to invent a new unit called the "gram-equivalent of force," which corresponds to the force exerted by one gram under the influence of gravity at the earth's surface, equal to 980 dynes.

Conveniently, though, our choice of units and the planet live on make the conversion rather easy. 1 gram equivalent tracking force is within 2% of 1000 dynes, making the conversion pretty easy: 1 gram is about 1000 dynes: simple. Throughout the following evaluation, we'll show the tracking force in dynes and gram-equivalent force when convenient.

One of the primary advantages of using consistent dimensions and units is that it provides a means of verifying that our work is proceeding correctly along sound lines. We can use a method called "dimensional consistency" which verifies that the units and dimensions we start with and the operations we perform on them give us not only the correct numerical results, but the correct units as well.

Let's look at an example of dimensional consistency. A spring and a mass can form a mechanical resonant system. We are familiar with the formula for the natural frequency of such a system:

$$F = \frac{1}{2\pi\sqrt{M \cdot C}}$$

where F is the frequency, M is the mass and C is the compliance. Well, frequency, mass and compliance don't tell us much, what we need is frequency, mass and compliance *in what units*. More specifically, the frequency is in units of Hz, or cycles per second, mass (in this context) is in grams, and compliance is in centimeters per dyne.

But how, you might ask, does putting mass and compliance together end up in frequency? Because of the correct use of the proper units, that's how.

Let's look at this in more detail. Again, mass is in grams and compliance is in centimeters per dyne. The product of these two, $M \cdot C$, combines the units and creates a new number in units of gram centimeters per dyne. Well, that seems even more complicated until remember the definition of a dyne: it's a gram centimeter per second squared. So, in equation form, we simply substitute the definition of a dyne:

$$\frac{g \cdot cm}{dyne} = \frac{g \cdot cm}{\frac{g \cdot cm}{s^2}}$$

Now, we can start cancelling out units that appear in both the numerator and denominator, since if the appear as factors in both places, the result is one: grams cancel out, centimeters cancel out, and we are left with $1/s^2$ in the denominator. That's the same as having s^2 in the numerator. Take the square root of that and you have simply time, in seconds.

Replace everything that used to be under the radical with a number in units of seconds, and we get:

$$\frac{1}{2\pi\, s}$$

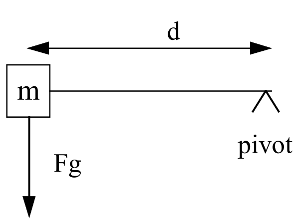
which is in units of "per second." The 2π is the number of radians per cycle. Thus, we have started with grams and centimeters per dyne and logically ended with cycles per second, or Hz.

If, instead of mass, you used weight (which, remember, is a force), you'd find that you'd end up in some weird set of dimensions that do not make logical sense (in this case, the result would be in the reciprocal of the square root of centimeter!).

By ensuring dimensional consistency and thus correct units, we can verify that our path is correct and we end up with results consistent with the physical system we are dealing with. We'll be using dimensional analysis through this and other articles.

3 BALANCE

In most high-quality tone arms, balance and tracking force is achieved by balancing masses relative to the pivot point. Gravity exerts forces on each mass of the tone arm, each of these forces in turn create their own torque around the pivot point.



Torque (designated as G) is a force around a pivot. The amount of torque is equal to the force times the distance from the pivot:

$$G = F \times d$$

The force, as mentioned, results from the acceleration of gravity on the mass:

$$F = m\, g$$

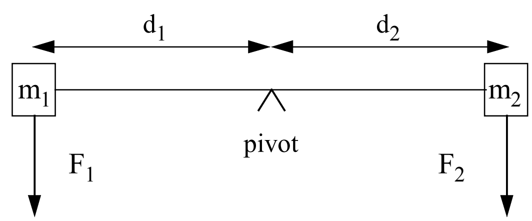
and g is the acceleration due to gravity, $980 \text{ cm}/\text{sec}^2$. As an example, say the mass m is 5 grams, and the distance d between that mass and the pivot is 20 cm (like a phono cartridge ate the end of a tone arm). The force exerted by the mass due to gravity is:

$$F = 5g \times 980\ cm/\sec^2 = 4900\ dynes$$

and the resulting torque is:

$$G = 4900\ dynes \times 20\ cm = 98{,}000\ dyne\ cm$$

In this case, we have a single source of torque, resulting in an unbalanced force. An unbalanced force means that the system will move. To achieve balance, we must have an equal amount of torque, but applied in the opposite direction. One way to do this is to have a mass of equal size an equal distance on the other side of the pivot:



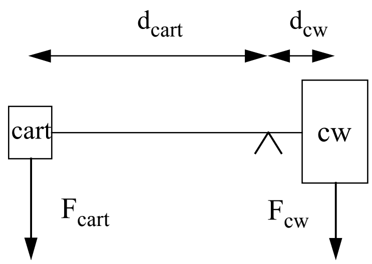
The total, or net torque on this system then is:

$$G_{NET} = F_1\, d_1 - F_2\, d_2$$

The minus sign on the second term is a result of the torque being applied in the opposite direction. Since $\mathrm{m}_1 = \mathrm{m}_2$, then $F_1 = F_2$. And since $\mathrm{d}_1 = \mathrm{d}_2$ then $F_1\, d_1 = F_2\, d_2$. If the two terms are equal, then the resulting torque is zero. This is the condition needed to achieve static balance.

Achieving static balance can be done with two equal masses equidistant from the pivot point on opposite sides of the pivot, but that is not the only way. All that is required to achieve static balance is the *net sum of all torques around the pivot is zero.*

Imagine, instead, on the right side of the pivot, we have a mass of 100 grams, but only 1 cm from the pivot. Imagine this arrangement for a simple, hypothetical tone arm:



The force exerted by that counterweight mass is:

$$F_{CW} = 100g \times 980cm/\sec = 98{,}000\ dynes$$

Spaced 1 cm from the pivot point, it exerts a torque of:

$$G_{CW} = 98{,}000\ dynes \times 1\ cm = 98{,}000\ dyne\ cm$$

And the force exerted by that cartridge mass is:

$$F_{CART} = 5g \times 980cm/\sec = 490\ dynes$$

Spaced 20 cm from the pivot point, it exerts a torque of:

$$G_{CART} = 490\ dynes \times 20\ cm = 98{,}000\ dyne\ cm$$

precisely equal to the torque exerted by counterweight on other side of the pivot.

The sum of these two torques, then, becomes:

$$G_{NET} = 98{,}000\ dyne\ cm - 98{,}000\ dyne\ cm = 0$$

The net torque is zero, thus the net forces are zero, and thus the arm is statically balanced.

3.1 Applying Tracking Force

To apply tracking force, the arm is unbalanced usually by moving the counterweight. This results in an unequal application of torque by the cartridge and the counterweight. If the counterweight is moved inwards towards the pivot, it's contributing torque is now reduced resulting in a net downwards force at the cartridge.

Let's look at this scenario in an analytical fashion. Take our example above, only now lets place the center of mass of the counterweight 0.8 cm from the pivot point, rather than 1 cm. The torque it now contributes is:

$$G_{CW} = 98{,}000\ dynes \times 0.8\,cm = 78{,}400\ dyne\ cm$$

The net torque then becomes:

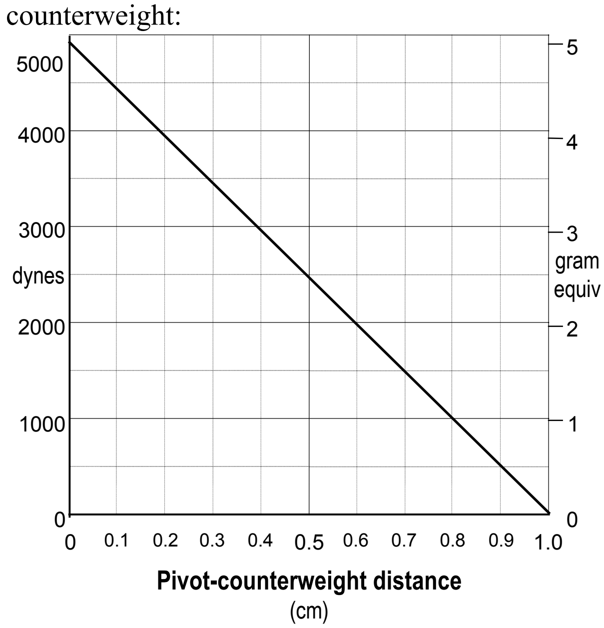
$$G_{NET} = 98{,}000\ dyne\ cm - 78{,}400\ dyne\ cm$$
$$G_{NET} = 19{,}600\ dyne\ cm$$

Then, at the stylus point, 20 cm from the pivot, that unbalanced torque translates into a downward force of:

$$F_{CART} = 19{,}600\ dyne\ cm \div 20\ cm$$
$$F_{CART} = 980\ dynes$$

And 980 dynes is the force equivalent of 1 gram. By moving the counterweight in a mere 2 mm (0.2 cm), we've generated 1 gram equivalent force of tracking weight at the stylus.

Now, if we look at the results above, we will see that the net forces resulting from unbalanced torque goes as a linear function of the distance of the counterweight:



This turns out to be very convenient for calibrating the tracking force. Imagine that the tone arm shaft behind the pivot is threaded, and that each turn of the thread is 1 mm apart. Similarly the counterweight is threaded onto this shaft. Rotating the counterweight back and forth on the thread moves the counterweight towards or away from the pivot. Each turn moves it 1 mm. But a 1mm change in the position of the tone arm results (in our example) of a change in force of ½ gram equivalent (490 dynes).

Now, adjust the counterweight so that the arm is in perfect balance. Want a gram force equivalent tracking weight of 1 gram? Easy, turn the counterweight on its thread exactly two turns, which moves it 2 mm (0.2 cm) towards the pivot. How about 1.5 gram equivalent force? That's three turns. The accuracy of the scheme depends only upon the accuracy of the thread and the weight of the counterweight. Both these quantities can be easily designed and controlled during manufacturing to a degree of precision that far exceeds what's necessary in this application. As a result, the tracking force calibrations on tone arms that apply tracking force by rotating a counterweight along a thread are usually very accurate, often more so than expensive, often friction-laden tracking force gauges. This is contrary to the popular wisdom in the high-end turntable world.

4 EFFECTIVE MASS

The subject of the effective inertial mass of tone arms is an important one, but one which seems to be sorely misunderstood, even by alleged "experts." As an example, one need only look at some of the claims made in the popular press. One episode in particular stands out: the head of a company making high-quality turntables, in an infamous letter to the editors of High Fi News and Record Review (ca 1976), stated categorically that tone arms *cannot* have mass, since everything is moving around a pivot, all that mass is really translated into moment of inertia. And without mass, the system can't have a mass-compliance resonance. Unfortunately, the author of this letter failed to get tone arms to subscribe to his theory, as they have behaved before and since in every way as if they did have mass.

This confused view of the mechanics of tone arms does, however, serve to highlight that the analysis of the effective mass is not necessarily a straightforward exercise. While not necessarily just simple linear mechanics. it is still well within the realm of high school physics.

4.1 Moments of Inertia

Rotary motion about a pivot point obeys precisely the same physical laws as linear motion, and can be analyzed in the same way. If a body of mass *m* is moving at a velocity *v*, it's kinetic energy *E* is:

$$E = \frac{1}{2} m v^2$$

This is true whether the body is moving in a straight line or whether it is moving in uniform circular motion around a pivot point. In the case of circular motion, we can say that the velocity *v* is equal to the angular velocity ω[1] and the distance from the pivot point *r*:

$$v = \omega r$$

Now, calculating the energy of such a body merely requires us to use ωr in place of *v*:

$$E = \frac{1}{2} m\, \omega^2 r^2$$

Let's rearrange this equation slightly to give us:

$$E = \frac{1}{2} \omega^2 \; m r^2$$

The quantity mr^2 is referred to as the *moment of inertia* of the object, and is an important property of all rotating masses. It's normally designated by the symbol *I*, and, for point objects of mass *m* that are *r* distant from the center of rotation, the moment of inertia is calculated as:

$$I = mr^2$$

and its units are in gram centimeters squared, or $g\ cm^2$.

Now, this works fine for point masses (those whose mass is concentrated over a distance that is small compared to *r*), but tone arms aren't made out of such objects. Just think about the tone arm tube itself, a tube of some material that has its mass distributed in possibly a complex fashion over the entire distance between the pivot and the cartridge. Some portions are close to the pivot, meaning that *r* is small, some portions are far away, and *r* is large. What is the contribution of such an extended object to the total moment of inertia?

Well, let's literally break the problem down and analyze it. Pretend our tone arm tube is a uniform tube 20 cm long weighing, say, 3 grams. If all the mass were concentrated at the end, far from the pivot, it's moment of inertia would be

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[1] ω is usually expressed in radians per second. A radian is the distance around the circumference of the circle equal to the radius of the circle. Consequently, there are 2π radians in each full circle or revolution.

$$I = 3g \times (20cm)^2$$
$$I = 3g \times 400cm^2, .$$
$$I = 1200\ g\ cm^2$$

On the other hand, if all the mass was concentrated at the pivot point, it's moment of inertia would be 0. Halfway along, it would be

$$I = 3g \times (10cm)^2$$
$$I = 3g \times 100cm^2, .$$
$$I = 300\ g\ cm^2$$

But, as we know, the mass is distributed uniformly along the length. Let's pretend the mass is concentrated into 11 point masses, each spaced every 2 cm along the arm from the pivot to the end. Point mass number 1, $\frac{3}{11}$ of a gram, is at the pivot point, point mass number 11, having the same mass as the first, is at the end. The total moment of inertia is simply the sum of each one:

$$I_{TOTAL} = I_1 + I_2 + \ldots + I_{11}$$

The moment of inertia of mass 1 is:

$$I_1 = \frac{3}{11} g \times (0\ cm)^2 = 0\ g\ cm^2$$

while that of mass 11 is:

$$I_1 = \frac{3}{11} g \times (20\ cm)^2 = 109\ g\ cm^2$$

In fact, we can generalize and say the moment of inertia of any segment is:

$$I_n = \frac{3}{11} g \times \left[2(n-1)\ cm\right]^2$$

and that the total moment of inertia (in more general terms and standard notation) is:

$$I_{TOTAL} = \sum_{n=1}^{11} \frac{3}{11} g\left[2(n-1)cm\right]^2$$

working this out, we end up with a figure of $420g\ cm^2$. We can break it up into smaller segments and see what the approximation works out to. With 20 segments, we get $410g\ cm^2$, 40, we end up with $405g\ cm^2$, 200 yields $401g\ cm^2$. And if we carry the experiment far enough, say dividing into 1,000,000 segments, we get something like $400.001\ g\ cm^2$.

In fact, we can also go look up in an mechanical engineering handbook and see what the moment of inertia is for a uniform thin hollow tube pivoted from one end, and we find:

$$I = \frac{ml^2}{3}$$

which agrees with our little approximation to a very high degree. While we're there, we'll see moment of inertia calculations for all sorts of shapes of objects. (I have in front of me a copy of Machinery's Handbook, published by The Industrial Press, wherein we find a table spanning several pages illustrating the moments of inertia of dozens of different shapes and profiles).

So, for completeness, we would include these calculations into the total. However, for simplicity, we'll simply confine ourselves to our simpler model.

The implications of all this are significant, in that it tells us that the energy storage is proportional to the mass, but to the *square* of the distance of that mass from the center of rotation. We can think of several intuitive examples of this property, Flywheels, as in the simple toy gyroscope, try to have all their mass concentrated as far from the pivot point as possible, to maximize energy storage.

These implication has important effect on tone arm design and performance. Let's take our simple tone arm from the previous example and examine what the implications of the placement of masses relative to the pivot have. Under the assumption that all the portions of our tone arm are rigidly connected together, let's calculate the total moment of inertia.

The moment of inertia of the cartridge is:

$$I_{CARTt} = 5g \times (20\ cm)^2$$
$$I_{CART} = 5g \times 400\ cm^2$$
$$I_{CART} = 2000\ g\ cm^2$$

while that for the counterweight is:

$$I_{CW} = 100g \times (1\ cm)^2$$
$$I_{CW} = 100g \times 1\ cm^2$$
$$I_{CW} = 100\ g\ cm^2$$

Hold on a second! The counterweight is 20 times the mass of the cartridge, yet its moment of inertia is 1/20th that of the cartridge. That's because, as we saw above, there's that dependency on the *square* of the distance from the pivot.

Remember that the total moment is simply the sum of each contributor, so the total is:

$$I_{TOTAL} = I_{CART} + I_{CW}$$
$$I_{TOTAL} = 2000g\ cm^2 + 100g\ cm^2$$
$$I_{TOTAL} = 2100g\ cm^2$$

What is important from this analysis is that for most tone arms, *the single largest contributor to the total moment of inertia is the cartridge's, not the counterweight's*.

And let's look at what first seems to be a completely counterintuitive result. Let's double the mass of the counterweight to 200 grams. But, remember, to achieve balance, we'll have to move it closer to the pivot, in this case from 1 cm to 0.5 cm. Now, the contribution to moment of inertia becomes:

$$I_{CW} = 200g \times (0.5\ cm)^2$$
$$I_{CW} = 200g \times 0.25\ cm^2$$
$$I_{CW} = 50\ g\ cm^2$$

And the total now becomes:

$$I_{TOTAL} = 2000g\ cm^2 + 50g\ cm^2$$
$$I_{TOTAL} = 2050g\ cm^2$$

Thus, the seemingly contradictory result that making the counterweight heavier reduces the total moment of inertia.

4.2 And Back to Effective Mass

Well, this moment of inertia all seems very interesting, but we are looking for the effective mass. How do we convert from total moment of inertia back to the effect mass?

The important missing part of the question is, how do we convert back to the effective mass at the *where*? The obvious answer, I hope, is at the stylus tip, 20 cm away from the pivot.

Remember that we got to the moment of inertia for a point mass by:

$$I = m\,r^2$$

What we want to know is what is the equivalent mass m at the radius r. That's simple (and it's the step that Tiffenbrun forgot to take in is incomplete analysis). Simple re-arrange the equation so that m is on one side. Divide both sides of the equation by r^2 and you get:

$$\frac{I}{r^2} = m$$

So, in our example, the arm has a total moment of inertia of $2100g\ cm^2$, so the resulting effective mass at the stylus tip, 20 cm away from the pivot, would be:

$$m_{EFF} = \frac{2100g\ cm^2}{(20\ cm)^2}$$
$$m_{EFF} = \frac{2100g\ cm^2}{400cm^2}$$
$$m_{EFF} = 5.25\ grams$$

And, as we see, the single largest contributor to the total effective mass is the cartridge, not the counterweight. Again, this is as a consequence of the dependency on the square of the distance from the pivot.

Let's look at our second example, with the 200 gram counterweight:

$$m_{EFF} = \frac{2050g\ cm^2}{(20\ cm)^2}$$
$$m_{EFF} = 5.125\ grams$$

Again, somewhat counter-intuitively, the arm with the *heavier* counterweight ended up with the *lower* effective mass. Not by a lot, but it's still a real effect.

This then contradicts the myth that to lower the mass of a tone arm, all elements must have their mass reduced. It further suggests that if you need to lower the effective mass of an arm, you're much better spending the effort at the end where the cartridge is, not near the pivot or counterweight.

4.3 Tracking Force and Effective Mass

One myth that I have heard repeatedly is that to get the total effective mass, you must add the tracking force to the normal effective mass. This would be true if and only if you got that tracking force simply by adding a mass right to the cartridge, sort of along the lines of the old trick of taping a penny to the headshell.

Most high-quality tone arms achieve positive tracking force by unbalancing the arm, as shown above, moving the counterweight closer to the pivot from the zero-balance position. We can calculate the effects this has on the effective mass.

In our example, we had to move our 100 gram counterweight to a position of 0.8 cm away from the pivot point to achieve a 1 gram equivalent tracking force at the stylus tip. Under that condition, the counterweight's contribution to the total moment of inertia is:

$$I_{CW} = 100g \times (0.8\ cm)^2$$
$$I_{CW} = 100g \times 0.64\ cm^2$$
$$I_{CW} = 64g\ cm^2$$

The total moment of inertia becomes

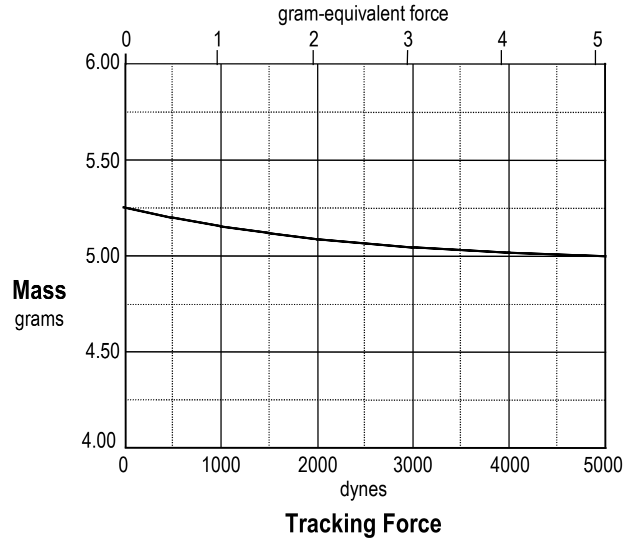
$$I_{TOTAL} = I_{CART} + I_{CW}$$
$$I_{TOTAL} = 2000g\ cm^2 + 64g\ cm^2$$
$$I_{TOTAL} = 2064g\ cm^2$$

And the resulting effective mass, 20 cm away from the stylus tip, becomes:

$$m_{EFF} = \frac{2064g\ cm^2}{400cm^2}$$
$$m_{EFF} = 5.16\ grams$$

So, quit contrary to the another of the popular myths, in tone arms that achieve tracking force by unbalancing the tone arm, the tracking force is not added to the effective mass. In fact, dialing in the tracking force moving the counterweight closer to the pivot from the zero-balance state *reduces* the effective tone arm mass.

Similar to the graph we saw above, we can produce a graph, given our hypothetical tone arm, of effective mass vs tracking force:



Again, the graph shows us the counter-intuitive effect of the mass decreasing with increasing tracking force, even though the dependence is small, at best. In our example, increasing the tracking force from 1000 to 5000 dynes, an increase of 500% or a factor of 5, results in a reduction of mass of only 3%, a negligible change, at the worst.

4.4 A More Complete Model

We have done most of our exploration making some simplifying assumptions: that the only objects contributing to the effective mass are the cartridge and the counterweight. Clearly, in practical tone arms, there's more than this. There is, at the very minimum. Some sort of mounting arrangement for the cartridge, the arm tube, and some sort of support for the counterweight. There is also whatever object at the pivot point that holds it all together, but, remember, if it's at the pivot point, its contribution to the effective mass is negligible

Let's apply some practical numbers and see what we might come up with for the effective mass for an actual tone arm. I have available a late model AR tone arm which is easily disassembled and measured. Here's the tally:

Item	Mass g	Distance cm*	Moment of Inertia g cm^2
Cartridge Mount	10.3	21	4540
Arm tube	10.8	18.7	1260
Counter weight	87	~4	1392
Counterweight mount	16	6.5	225
Internal wires	0.4	22	64.5
Signal terminals	1.1	22	532
Total	125.6g		8014 g cm^2

* The distance is distance from the pivot to the center of mass of the object for objects such as the cartridge mount, or the length of the object for the tone arm tube, internal wiring, etc.

The stylus-pivot distance of this arm is about 22.5 cm, so that the effective mass of the arm, exclusive of the cartridge or its mounting hardware, is $\frac{8014\ g\ cm^2}{(22.5\ cm)^2} = 15.8g$. This is neither a particularly high-mass nor a particularly low-mass tone arm. If one wanted to, say, reduce the mass of this arm, there are several approaches. You could increase the mass of the counterweight, getting it closer to the pivot, but that would not do much for you, since the counterweight contributes only 17% of the total effective mass as it is. The biggest gain would be in reducing the amount of mass in the cartridge mounting plate. With some careful shaving, I would see removing 5 grams of material here. That's 5 grams in effective mass, since it's at the end of the tone arm.

Tone arms with masses approaching 5 grams or less are rare and often involve some significant compromises in strength and rigidity. Those with masses much exceeding 20 grams significantly limit the range of cartridges that can be used to ones with low compliances.

4.5 Arm Mass and Resonance

The significance of the effective mass in relation to the stylus compliance comes from the fact that the two, together, form a classic mechanical resonant system. It is the combination of the effective mass and the compliance that determine the frequency of that resonance:

$$F = \frac{1}{2\pi\sqrt{M_{EFF} \cdot C_{STYLUS}}}$$

where M_{EFF} is the total effective mass of the arm-cartridge system, C_{STYLUS} is the mechanical compliance of the stylus suspension, and F is the resonant frequency of system. As long as we maintain consistent units, we can simply plug the numbers in and out comes the resonant frequency in Hertz.

Looking at our example arm above, the AR, combined with a 5 g cartridge gives a total effective mass of 21g. Looking at typical moving-magnet compliance, we find compliance figures of $10 \times 10^{-6}\ cm / dynes$, that is for every dyne of force applied to the stylus, it will deflect 10 millionths of a centimeter as a result. That means, for example, that if the tracking force is set to 1 gram equivalent force, or 980 dynes, the stylus deflection will be

$$x = C \times F$$

$$x = 10 \times 10^{-6}\ cm / dyne \times 980\ dyne$$

$$x = 9.8 \times 10^{-3}\ cm$$

or about 0.1 mm.

The resonant frequency of such a system will be:

$$F = \frac{1}{2\pi\sqrt{20.8g \cdot 10 \times 10^{-6}\ cm / dyne}}$$

$$F = \frac{1}{2\pi\sqrt{20.8 \times 10^{-4}\ g\ cm / g\ cm / s^2}}$$

$$F = \frac{1}{2\pi \cdot 1.44 \times 10^{-2}\ s}$$

$$F = 11\ s^{-1}$$

or 11 Hz.

11 Hz is about ideal for a cartridge/arm resonant frequency, being just about in the "ideal" range of 7 to 12 Hz. If the effective mass of the arm/cartridge were substantially higher, say, 42 grams, the resonant frequency would be lower, less than 8 Hz. One the other hand, using one of the very high-compliance cartridges, one with a compliance of, say, $40 \times 10^{-6}\ cm / dyne$, would lead to a lower resonance as well, in this case, about 5.5 Hz.

Why the resonance should be within these confines is beyond the scope of this article, but suffice it to say that we are bounded at the top end by the risk of intruding into the musical content of the record, and at the low end by mechanical interference due to external vibrations, mis-tracking due to warp-induced movements, and so on.

5 CONCLUSION

We've investigated two area of fundamental mechanics as they apply to tone arm properties: the application of lever arms and torque to determine tracking weight in statically weight-balanced tone arms and the use of concepts of moment of inertia in evaluating the effective mass of tone arms. We have seen that, especially in the evaluation of effective mass, some of the intuitive assumptions are wrong regarding how each element of the arm contributes to the effective mass. For example, we see that even though it is the most massive object in a tone arm, the counterweight, is actually only a minor contributor to the total moving mass. One of the most important concepts to be grasped is that *tracking force is not effective mass*.

We explored how the cartridge mass and stylus suspension compliance interact to form a resonant system and how that resonance can be calculated.

We have also seen that the claim that "the tracking force must be added to the effective mass is simply not supportable.

The LP still enjoys a strong and loyal following in some niche markets. While it has clearly dropped out of the mainstream of the music delivery market, there are those that are devoted to its survival. From a practical standpoint, there are still many recordings and significant performances that are simply not available on alternate media (CD, cassette, etc.), meaning that there will, for some time to come, be a need for correct playback of these records. I hope that this discussion, a better understanding of those principals underlying record reproduction can be obtained.